

Chapter 12 Benefit-Cost Analysis

12.1)

$$\begin{aligned}B &= \$117,400(P / A, 6\%, 5) \\&= \$494,535.76 \\C &= \$5,000 + \$48,830(P / A, 6\%, 5) \\&= \$210,691.49\end{aligned}$$

$$\begin{aligned}\text{BC}(6\%) &= \frac{\$494,535.76}{\$210,691.49} \\&= 2.35 > 1\end{aligned}$$

This project is justifiable based on the benefit-cost analysis.

12.2)

$$\begin{aligned}B &= \$250,000(P / A, 6\%, 25) + \$50,000(P / F, 6\%, 25) \\&= \$3,207,500 \\C &= \$1,200,000 + \$100,000(P / A, 6\%, 25) \\&= \$2,478,340\end{aligned}$$

$$\begin{aligned}\text{BC}(6\%) &= \frac{\$3,207,500}{\$2,478,340} \\&= 1.29 > 1\end{aligned}$$

12.3)

(a) BC(*i*) analysis:

- Design A:

$$\begin{aligned}I &= \$400,000 \\C' &= \$50,000(P / A, 8\%, 15) = \$427,974 \\B &= \$85,000(P / A, 8\%, 15) = \$727,557.5 \\ \text{BC}(8\%) &= \frac{B}{I + C'} \\&= \frac{\$727,557.5}{\$400,000 + \$427,974} \\&= 0.88 < 1\end{aligned}$$

- Design B:

$$I = \$300,000$$

$$C' = \$80,000(P/A, 8\%, 15) = \$684,758$$

$$B = \$85,000(P/A, 8\%, 15) = \$727,557.5$$

$$\begin{aligned} BC(8\%) &= \frac{B}{I + C'} \\ &= \frac{\$727,557.5}{\$300,000 + \$684,758} \\ &= 0.74 < 1 \end{aligned}$$

- Incremental analysis: Fee collections in the amount of \$85,000 will be the same for both alternatives. Therefore, we will not be able to compute the $BC(i)$ ratio. If this happens, we may select the best alternative based on either the least cost ($I + C'$) criterion or the incremental $B'C(i)$ criterion.

Using the incremental $B'C(i)$ criterion,

$$\begin{aligned} \Delta B'C(8\%)_{A-B} &= \frac{\Delta B - \Delta C'}{\Delta I} \\ &= \frac{0 - (\$427,974 - \$684,758)}{\$100,000} \\ &= 2.56 > 1 \end{aligned}$$

Select Design A.

(b) Incremental analysis (A-C):

$$\begin{aligned} \Delta B'C(8\%)_{A-C} &= \frac{\Delta B - \Delta C'}{\Delta I} \\ &= \frac{0 - (\$427,974 - \$556,366)}{\$50,000} \\ &= 2.57 > 1 \end{aligned}$$

Select Design A.

12.4)

- Building X:

$$\begin{aligned}
B_X &= \$1,960,000(P / A, 10\%, 20) \\
&= \$16,686,585 \\
C_X &= \$8,000,000 + \$240,000(P / A, 10\%, 20) \\
&\quad - \$4,800,000(P / F, 10\%, 20) \\
&= \$9,329,766 \\
BC(10\%)_X &= \frac{\$16,686,585}{\$9,329,766} \\
&= 1.79 > 1
\end{aligned}$$

- Building Y:

$$\begin{aligned}
B_Y &= \$1,320,000(P / A, 10\%, 20) \\
&= \$11,237,904 \\
C_Y &= \$12,000,000 + \$180,000(P / A, 10\%, 20) \\
&\quad - \$7,200,000(P / F, 10\%, 20) \\
&= \$12,462,207 \\
BC(10\%)_Y &= \frac{\$11,237,904}{\$12,462,207} \\
&= 0.90 < 1
\end{aligned}$$

Since Building Y is not desirable at the outset, we don't need an incremental analysis. Building X becomes the better choice.

12.5) Incremental $BC(i)$ analysis:

Present worth	Proposals			Incremental	
	A1	A2	A3	A3-A1	A2-A1
I	\$100	\$300	\$200	\$100	\$200
B	\$400	\$700	\$500	\$100	\$300
C'	\$100	\$200	\$150	\$50	\$100
$B'C(i)$	3	1.7	1.75	0.50	1

Select either A1 or A2.

12.6) Incremental *BC* analysis:

	Equivalent Present Worth				
	Design			Incremental	
	Design A	Design B	Design C	C-B	A-B
<i>B</i>	\$7,824	\$7,070	\$5,656	-\$1,414	\$754
<i>I</i>	\$2,440	\$880	\$1,600	\$720	\$1,560
<i>C'</i>	\$3,865	\$3,394	\$2,922	-\$472	\$471
BC(10%)	1.24	1.65	1.25	-5.7	0.37

Select Design B.

12.7)

(a) The benefit-cost ratio for each alternative:

- Alternative A:

$$\begin{aligned}
 B &= (\$1,000,000 + \$250,000 + \$350,000 \\
 &\quad + \$100,000)(P/A, 10\%, 50) \\
 &= \$16,855,185
 \end{aligned}$$

$$\begin{aligned}
 C &= \$8,000,000 + \$200,000(P/A, 10\%, 50) \\
 &= \$9,982,963
 \end{aligned}$$

$$BC(10\%)_A = 1.69 > 1$$

- Alternative B:

$$\begin{aligned}
 B &= (\$1,200,000 + \$350,000 + \$450,000 \\
 &\quad + \$200,000)(P/A, 10\%, 50) \\
 &= \$21,812,592
 \end{aligned}$$

$$\begin{aligned}
 C &= \$10,000,000 + \$250,000(P/A, 10\%, 50) \\
 &= \$12,478,704
 \end{aligned}$$

$$BC(10\%)_B = 1.75 > 1$$

- Alternative C:

$$B = (\$1,800,000 + \$500,000 + \$600,000 + \$350,000)(P/A, 10\%, 50) = \$32,223,147$$

$$C = \$15,000,000 + \$350,000(P/A, 10\%, 50) = \$18,470,185$$

$$BC(10\%)_C = 1.74 > 1$$

(b) Select the best alternative based on $BC(i)$:

$$BC(10\%)_{B-A} = \frac{\$21,812,592 - \$16,855,185}{\$12,478,704 - \$9,982,963} = 1.99 \text{ (Select B.)}$$

$$BC(10\%)_{C-B} = \frac{\$32,223,147 - \$21,812,592}{\$18,470,185 - \$12,478,704} = 1.74 \text{ (Select C.)}$$

Select C.

Comments: You could select the best alternative based on $B'C(i)$:

	A	B	C
I	\$8,000,000	\$10,000,000	\$15,000,000
C'	\$1,982,963	\$2,478,704	\$3,470,185
$B'C(10\%)$	1.86	1.93	1.92

$$B'C(10\%)_{B-A} = \frac{(\$21,812,592 - \$16,855,185) - (2,478,704 - 1,982,963)}{\$10,000,000 - \$8,000,000} = 2.23 \text{ (Select B.)}$$

$$B'C(10\%)_{C-B} = \frac{(\$32,223,147 - \$21,812,592) - (\$3,470,185 - \$2,478,704)}{\$15,000,000 - \$10,000,000} = 1.88 \text{ (Select C.)}$$

12.8)

- Option 1-The “long” route:

Users’ annual cost = 22 miles × \$0.25 per mile × 400,000 cars

$$= \$2,200,000$$

Sponsor’s annual cost = \$21,000,000(A / P, 10%, 40) + \$140,000

$$= \$2,287,448$$

- Option 2-Shortcut:

Users’ annual cost = 10 miles × \$0.25 per mile × 400,000 cars

$$= \$1,000,000$$

Sponsor’s annual cost = \$45,000,000(A / P, 10%, 40) + \$165,000

$$= \$4,766,674$$

- Incremental analysis (option 2-option 1):

Incremental users’ benefit = \$2,200,000 – \$1,000,000

$$= \$1,200,000$$

$$BC(10\%)_{2-1} = \frac{\$1,200,000}{\$4,766,674 - \$2,287,448}$$

$$= 0.48 < 1$$

Assuming no do-nothing alternative, select option 1.

12.9) Multiple alternatives:

Projects	PW of Benefits	PW of Costs	Net PW	B/C ratio
A1	\$40	\$85	-\$45	0.47
A2	\$150	\$110	\$40	1.36
A3	\$70	\$25	\$45	2.80
A4	\$120	\$73	\$47	1.64

Since the BC ratio for project A1 is less than 1, we eliminate it from our comparison. Incremental Analysis: ordering (A3, A4, A2)

- A3 versus A4:

$$BC(i)_{A4-A3} = \frac{\$120 - \$70}{\$73 - \$25}$$

$$= 1.04 > 1$$

Select A4.

- A2 versus A4:

$$\begin{aligned} BC(i)_{A2-A4} &= \frac{\$150 - \$120}{\$110 - \$73} \\ &= 0.81 < 1 \end{aligned}$$

Select A4.

12.10) Given $i = 8\%$, $g = 10\%$, garbage amount/day = 300 tons

- (a) The operating cost of the current system in terms of \$/ton of solid waste:

- Annual garbage collection required (assuming 365 days):
Total amount of garbage = 300 tons \times 365 days
= 109,500 tons/year
- Equivalent annual operating and maintenance cost:

$$\begin{aligned} PW(8\%) &= -\$905,400(P / A_1, 10\%, 8\%, 20) \\ &= -\$20,071,500 \\ AEC(8\%) &= \$20,071,500(A / P, 8\%, 20) \\ &= \$2,044,300 \end{aligned}$$

- Operating cost per ton:

$$\begin{aligned} \text{cost per ton} &= \frac{\$2,044,300}{109,500} \\ &= \$18.67 \text{ ton} \end{aligned}$$

- (b) The economics of each solid-waste disposal alternative in terms of \$/ton of solid waste:

- Site 1:

$$\begin{aligned} AEC(8\%)_1 &= \$4,053,000(A / P, 8\%, 20) \\ &\quad + \$342,000(P / A_1, 10\%, 8\%, 20)(A / P, 8\%, 20) \\ &\quad - (\$13,200 + \$87,600) \\ &= \$1,084,773.719 \\ \text{Cost per ton} &= \$1,084,773.719 / 109,500 \\ &= \$9.91 \text{ per ton} \end{aligned}$$

- Site 2:

$$\begin{aligned}
 AEC(8\%)_2 &= \$4,384,000(A/P, 8\%, 20) \\
 &\quad + \$480,000(P/A_1, 10\%, 8\%, 20)(A/P, 8\%, 20) \\
 &\quad - (\$14,700 + \$99,300) \\
 &= \$1,417,042.61 \\
 \text{Cost per ton} &= \$1,417,042.61 / 109,500 \\
 &= \$12.94 \text{ per ton}
 \end{aligned}$$

- Site 3:

$$\begin{aligned}
 AEC(8\%)_3 &= \$4,764,000(A/P, 8\%, 20) \\
 &\quad + \$414,000(P/A_1, 10\%, 8\%, 20)(A/P, 8\%, 20) \\
 &\quad - (\$15,300 + \$103,500) \\
 &= \$1,301,871.57 \\
 \text{Cost per ton} &= \$1,301,871.57 / 109,500 \\
 &= \$11.89 \text{ per ton}
 \end{aligned}$$

- Site 4:

$$\begin{aligned}
 AEC(8\%)_4 &= \$5,454,000(A/P, 8\%, 20) \\
 &\quad + \$408,000(P/A_1, 10\%, 8\%, 20)(A/P, 8\%, 20) \\
 &\quad - (\$17,100 + \$119,400) \\
 &= \$1,340,928.66 \\
 \text{Cost per ton} &= \$1,340,928.66 / 109,500 \\
 &= \$12.25 \text{ per ton}
 \end{aligned}$$

Site 1 is the most economical choice.

(c) Incremental BC analysis:

	Present System	Site1	Site2	Site3	Site4
<i>B</i>	0	\$989.67	\$1,119.26	\$1,166.39	\$1,340.17
<i>I</i>	0	\$4,053.0	\$4,384.0	\$4,764.0	\$5,454.0
<i>C'</i>	\$20,071.48	\$7,581.68	\$10,640.95	\$9,177.82	\$9,044.81
Reduction in <i>C'</i> over the present system		\$12,489.80	\$9,430.53	\$10,893.66	\$11,026.67

- Site 1 vs. Site 2:

$$\begin{aligned}\Delta BC(8\%)_{2-1} &= \frac{(\$1,119.26 + \$9,430.53) - (\$989.67 + \$12,489.80)}{\$4,384 - \$4,053} \\ &= \frac{-\$2,929.68}{331} \\ &= -8.85 < 1 \\ &\text{Select Site 1.}\end{aligned}$$

- Site 1 vs. Site 3:

$$\begin{aligned}\Delta BC(8\%)_{3-1} &= \frac{(\$1,166.39 + \$10,893.66) - (\$989.67 + \$12,489.80)}{\$4,764 - \$4,053} \\ &= \frac{-\$1,419.42}{711} \\ &= -1.996 < 1 \\ &\text{Select Site 1.}\end{aligned}$$

- Site 1 vs. Site 4

$$\begin{aligned}\Delta BC(8\%)_{4-1} &= \frac{(\$1,340.17 + \$11,026.67) - (\$989.67 + \$12,489.80)}{\$5,454 - \$4,053} \\ &= \frac{-\$1,112.63}{\$1,401} \\ &= -0.7942 < 1 \\ &\text{Select Site 1.}\end{aligned}$$

The ultimate choice: Site 1.

12.11) **Suggestions:** Ask students to visit the Atlanta Airport website

(<http://www.atlanta-airport.com>) to obtain the current and projected airport operational statistics such as number of aircraft landings/takeoffs and passengers. If we just focus on some of the primary benefits and costs, we may identify the following elements:

- Sponsor's costs: (1) Required capital investments in airport expansion, (2) Additional O&M costs associated with the expanded airport operation.
- Sponsor's Revenue: (1) Incremental landing/takeoff fees due to additional traffic volumes, and (2) Increased parking and concession revenues due to additional passenger traffics.
- Users' Benefits: (1) Savings due to reduced waiting costs (value of travel time) (2) Savings on fuel costs for airliners due to reduced taxiing, landing and departure times, (3) Reduced air and noise pollution
- Users' Disbenefits: (1) Relocation of residents and commercial buildings due to airport expansion

Once these values are quantified, we compute the following for each option:

- Step 1: Users' net benefits = Users' benefits - Users' disbenefits.
- Step 2: Sponsor's net costs = Sponsor's costs - Sponsor's revenue.

Then, identify the option(s) with Users' net benefits exceeding the sponsor's net cost. Select the option with the largest differential net benefits. If the initial analysis based on the primary benefits and costs does not lead to any clear-cut choice, the analysis may be broadened to include the secondary benefits such as the regional economic impact studies.